Indian J. Phys. **67A** (6), 531 – 540 (1993)

Weak interaction rates for type-II supernovae

K Kar
Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Calcutta - 700 064, India

Abstract : The theory of type II supernovae over the years has developed into a detailed theory incorporating various aspects of nuclear and high energy physics. We give here a brief review of some of the areas of this theory where the rates of various weak interaction processes play key roles

Keywords : Weak interaction rates, supernovae, collapsed stars, neutrinos

PACS Nos. : 97 60.Bw, 23.40 –s, 13 15.–f

1. Introduction

The study of supernova explosions observed in different galaxies and over various wavelengths is one of the most exciting problems of present day observational astronomy. Alongwith that in the last two decades or so, there has been considerable progress in the theoretical understanding, both of type I and type II by using methodologies of different branches of physics-astrophysics, nuclear physics, high energy physics and statistical mechanics. The observation of SN1987A and the detection of neutrinos at 3 detectors simultaneously gave a big boost to this. It made the proposed theoretical model for supernovae real and convinced people about the central role played by neutrinos in the transport of energy both before and after bounce. Weak interaction rates *i.e.* electron capture as well as beta decay rates of the nuclei and neutrino interactions with matter of various forms play important roles at different stages of supernova evolution. This article attempts to review the areas where weak interactions control the evolution and describes the different model calculations. This will demonstrate the intimate relationship of different simple concepts of nuclear physics with the astrophysical problem of SN evolution. In Section 2 we give a brief introduction to the theory of type II supernovae. In Section 3 we describe why one needs to calculate the beta decay rates of some nuclei with $A > 60$ for presupernova evolution and efforts in that direction. Section 4 is devoted to establishing the importance of the evaluation of electron capture rates for gravitational collapse. Section 5 discusses the inverse beta decay process of neutrino capture and how the delayed mechanism depends sensitively on this. Finally, in Section 6 we discuss some high energy physics issues which have gained importance after the observation of SN1987A.

2. Type II supernovae–an overview

The presupernova evolution of massive stars involving nuclear burning ends after the completion of silicon burning when the central density reaches 10^9 to 10^{10} g/cc. For core masses greater than the Chandrasekhar mass, a gravitational collapse follows initiated by a combination of electron capture and iron photodissociation [1]. The rest of the evolution is completed in a few seconds and involves physics of electron capture, neutrino diffusions, dense matter equation of state and general relativity. The temperature T to start with is 0.6 to 0.8 MeV and the electron fraction Y_e (*i.e.* total number of electrons /total number of baryons) is about 0.42. The pressure at this stage has only the relativistic eleotron gas component and it turns out that the changes in Y_e play a crucial role in deciding the fate of the star. The important physics in the first stage of the collapse comes from electron capture on nuclei alongwith electron capture on a small fraction of protons. The neutrinos released escape the star cooling it in the process and reduces the total lepton fraction. At a density near 10^{12} g/cc , the neutrino mean free path becomes equal to the radius of the core and the neutrinos produced in the e-capture processes get trapped. Beyond this point there is no decrease in the total lepton fraction Y_e but only conversion from electrons to neutrinos (which also stops at a higher density when chemical equilibrium is established). The entropy per nucleon starting with the value of about 1k changes slowly and the nuclei get heavier but do not get broken up. Eventually the density becomes as high as nuclear matter density and then baryonic pressure starts dominating over the leptonic contribution. Matter becomes much stiffer and the collapse is followed by a hydrodynamical rebound of the inner core [2]. This bounce drives a shock wave rapidly outwards and for smaller stars (core mass < $1.35 M_O$), some simulations with low initial entropy and soft equations of state show the shock coming out of the core resulting in the ejection of matter [3-4]. This is called the prompt mechanism. But for most simulations the shock loses energy very fast due to nuclear dissociation and neutrino losses and stalls soon (a few mulliseconds) after its generation. Then the second mechanism called the 'delayed neutrino heating' operating on a much longer timescale of seconds is seen to revive the shock in some cases [5-6]. During the bounce and after, the central high density region of the core is opaque to neutrinos and is called the neutrinosphere. From the surface of that a very high flux of neutrinos is emitted. Due to very low strength of interaction of neutrinos with matter, only a small fraction of the emerging neutrinos interact with the matter ahead of the stalled shock. But in some calculations even that is seen to be enough to revive the shock over the long timespan. As only a few percent of the gravitational energy released is needed to account for the kinetic energy of ejected matter, the problem of theoretical modelling turns out to be a difficult job and each process has to be examined in a detailed fashion to arrive at the right theory. We discuss some of these aspects in the next few sections.

3. Beta decay rates for pre-supernova evolution

Beta decay and electron capture of neutron-rich nuclei play important roles in determining presupernova core structure [7] which in turn is important for the later evolution through gravitational collapse and shock formation. Complete shell model calculations are now available for nuclei with $A < 40$ with *sd*-shell interactions [8] which are highly successful in reproducing a large mass of nuclear data including spectra and excitation strengths. For heavier nuclei with A upto 60, Fuller Fowler and Newman (FFN) [9] calculated detailed estimates of Gamow-Teller resonance strengths and energy centroids which together with experimental energy levels and *ft*-values were used to compute reaction rates for a broad range of nuclei as well as stellar density and temperatures. But for nuclei with $A > 60$ very little information about the weak interaction rates were available till recntly. Microscopic shell model calculations are difficult to perform for the *fp*-shell nuclei because of the very large dimensions of the configuration spaces encountered. Also in contrast to the *sd*-shell there is a large uncertainty in the knowledge of the interaction and almost all of them are unsatisfactory in reproducing the experimental data accurately. On the other hand one now believes that the nuclei with $A > 60$ should be included in the pre-supernova evolution calculations because though they may have low abundances they have large decay rates (arising mainly from the high Q-values). Rough estimates find $dY e/dt \simeq 1.3 \times 10^{-6}\ s^{-1}$ while the contraction phase during Si burning lasts for about $4 \times 10^{4} s$ [10]. But only a selfconsistent calculation including these nuclei can give a proper estimate of Y_e, which controls the pressure and later determines the energy of the shock formed after the collapse.

In contrast to free decay of the nuclei which determines the half-life, these beta decays are in stellar conditions of non-zero temperature and in the presence of an electron gas where the liberated electron has to go to the top of the Fermi sea. The beta decay rates in this case is given by

$$\gamma = \ln 2 . \frac{(6250\,\mathrm{sec})^{-1}}{G} \sum_i (2J_i + 1) \exp(-E_i/kT)$$

$$\times \sum_f \left[B_F(E_f) + \left(\frac{g_A}{g_V}\right)^2 B_{GT}(E_f) \right] + f(Q - E_f). \tag{1}$$

Here, i sums over the initial states of the mother nucleus with energy E_i and spin J_i and f sums over the final states of the daughter nucleus at energy E_f. B_F and B_{GT} are the B-values of the Fermi and Gamow-Teller operators and the partition function G is given by $G = \sum_i (2J_i + 1) \times \exp(-E_i / kT)$. g_v and g_A are the vector and axial-vector coupling constants. As the Fermi resonance lies around the Isobaric Analogue State (IAS), much higher than the energy region allowed by the Q-values, $B_F(E_f)$ is zero for all available final states. The phase factor f in eq. (1) is given by

$$f(T,\mu_e,\varepsilon) = \frac{\int_1^{\epsilon_0} F(Z,\epsilon)\ \epsilon(\epsilon^2-1)^{1/2}\ (\epsilon_0-\epsilon)^2 d\epsilon}{i\ +\ \exp\{(\mu_e-\epsilon)/kT\}} \tag{2}$$

Wheré thė stellar temperature is T. μ_e is the electron chemical potential ane E_0 is the maximum energy available to the electron during decay, ϵ_0 $\left(= E_0/mc^2\right)$ and ϵ are the electron energies in mass units.

Aufderheide *et al* [11] in their calculation of the rates extended the FFN methodology for finding the nuclear matrix elements to nuclei with $A > 60$. They used the observed log ft values for some nuclei and extended them by some plasuability arguments to others. For the Fermi function $F(Z, \epsilon)$, they used the forms constructed by Schenter and Vogel by fitting simple analytical approximations. Their calculations cover both beta decay and electron capture rates but the calculations include transition only from the lowest state of the parent nucleus having an allowed transition and thus underestimate the total transition strength. They conjecture that though because of the complicated feedback mechanism involved, only a complete calculation can give the final answer but at densities of 10^8 g/cc the decay of some cobalt isotopes (in particular ^{63}Co) may be quite important.

Kar *et al* [12] used a statistical model to calculate the beta decay rates for a number of neutron rich nuclei with $A > 60$. It assumes the number of final states large enough to replace the sum over final states in eq.(2) by an integration . The Gamow-Teller (GT) sum rule strength is evaluated by a simple expression based on spectral distribution theory [13–14] involving neutron particle and proton hole occupancies. The GT strength distribution centroid is obtained from experimental data on (p, n) reactions and its form is assumed a Gaussian whose width is left as a free parameter and determined through the best fit of the calculated halflives of the free decays of a number of nuclei to their experimental values . Table 1 shows the comparison of the calculated halflives with the experimental ones alongwith the predictions by other methods. This method includes the contribution from the excited state of the mother nucleus and observes that for some nuclei at high densities (like 10^9 g/cc) it can increase the rates by an order of magnitude. Table 2 gives the rates for density 10^8 g/cc for temperatures 3,4 and 5 times 10^9 K for the nuclei ^{64}Co, ^{63}Co and ^{62}Co and then compares them with the values given by Aufderheide *et al* [11].

4. Electron capture during gravitational collapse

At the onset of collapse the core consists primarily of iron group nuclei. Using the equation of state one can find the mean nucleus and the relative abundances of different nuclei in the nuclear statistical equilibrium as a function of the density and temperature. Ones sees through these calculations that as the collapse proceeds and the density rises, the mean nucleus becómes heavier. For the collapse BBAL used a simple collapse rate given by

Table 1. Comparison of calculated and experimental halflives.

Nucleus	Experimental	Halflife $\tau_{1/2}$ (sec) Calculated		
		This work	Gross theory [30]	QRPA [31]
^{69}Cu	180	251.4		
^{68}Cu	31	22.0		
^{66}Cu	306	305.5		
^{67}Ni	21	47.0	94	23
^{65}Ni	9072	589.0		
^{62}Co	90	15 1		
^{63}Co	27.4	52.1		
^{64}Co	0.3	3.53		10.0
^{65}Co	1.25	5.66	8	8.59
^{61}Fe	360	34.5		
^{62}Fe	68	183.4		
^{63}Fe	4.9	3.49	10	14.8
^{62}Mn	0.88	1.05	2	0.773

Table 2. The beta decay rates for the nuclei ^{64}Co, ^{63}Co and ^{62}Co at $\rho = 10^8$ g/cc ($Y_e = 0.47$) Methods A is due to Kar *et al* [12], [32] where the rates include excited states of the parent with the numbers in the parenthesis the rates from the ground state only. Method B is due to Aufderheide *et al* [11] which gives the rates from the ground state only.

Nucleus	Method	Temperature (in °K)		
		3×10^9	4×109^9	5×10^9
^{64}Co*	A	0.804	0.854	0.901
		(0.698)	(0.699)	(0.701)
	B	2.08	2.09	2.09
^{63}Co	A	9.66×10^{-3}	1.10×10^{-2}	1.33×10^{-2}
		(9.31×10^{-3})	(9.60×10^{-3})	(9.89×10^{-3})
	B	1.53×10^{-2}	1.60×10^{-2}	1.69×10^{-2}
^{62}Co	A	4.33×10^{-2}	4.66×10^{-2}	5.01×10^{-2}
		(3.82×10^{-2})	(3.86×10^{-2})	(3.90×10^{-2})
	B	1.14×10^{-2}	1.89×10^{-1}	2.55×10^{-1}

*The reported values for ^{64}Co are for a negatively skewed distribution of the GT strength with an Edgeworth expansion of Skewness $\gamma_1 = -0.3$ and σ_N (the width due to nuclear forces) = 7.5 MeV, as with the incorporation of these one gets an improved halflife of 0.907s.

$$\frac{d}{dt}(\ln\rho) = 100\rho_{10}^{1/2}, \tag{3}$$

where ρ_{10} stands for the density in 10^{10} g/cc. Based on such collapse rates one-zone collapse calculations are available which give the evolution of the thermodynamic quantities like temperature, pressure, entropy etc.[15-16]. For these calculations one needs the capture rates on the nuclei with $A>56$. The decrease in the electron fraction is calculated by

$$\frac{dY_e}{dt} = -\lambda_{fp}X_p - \lambda_H X_H/A, \tag{4}$$

where λ_{fp} and λ_H are the capture rates and X_P and X_H are the fractions of free proton and heavy nuclei respectively. Fuller [15] pointed out an important effect called 'blocking' of the capture rates which shows that these rates become smaller by orders of magnitude once the neutron number (N) of the nuclei becomes greater than 40. This can be understood using simple zero order shell model (where one just fills up the single particle orbits). In this picture for a typical nucleus ^{74}Ge with $N = 42$, the neutron orbits $f_{7/2}$, $f_{5/2}$, $p_{3/2}$ and $p_{1/2}$ are all full and $g_{9/2}$ has 2 particles , whereas the valence protons are in $f_{7/2}$ and $f_{5/2}$ orbits. Thus the protons through e-capture cannot go to neutron orbits without a change in the l value and hence 'allowed' capture cannot take place. As for lighter nuclei, 'allowed' capture of the Gamow - Teller type makes the major contributions to the rates. This effect is called the 'neutron shell blocking' phenomenon. But in the realistic situations the rates do no really fall to zero. The 'unblocking' mechanism is firstly due to thermal effects which is proportional to $e^{-E/kT}$ where E is the energy required to lift a neutron from the $f_{5/2}$ orbit to $g_{9/2}$ orbit and T is the temperature. The other unblocking is through forbidden type transitions where Δl for the single particle transition need no longer be zero and protons can go from $f_{5/2}$ or $f_{7/2}$ to $g_{9/2}$ orbits.Calculations [15],[17-18] show the rates through these processes are still an order of magnitude lower than the allowed capture rates. Detailed hydrodynamical calculations have been performed using the capture rates [19] to estimate the final value of the electron fraction $\left(Y_e^f\right)$ As the shock energy E_{shock} is given by[15]

$$E_{\text{shock}} \sim \left(Y_e^f\right)^{10/3}\left(Y_e^i - Y_e^f\right), \tag{4a}$$

a proper evaluation of Y_e^f is very important. Calculations using a variable nuclear reaction network involving a number of nuclei (around the mean one) with different capture rates in conjunction with a gravitational collapse code, may be a better way of treating this problem and efforts are on in that direction.

5. Neutrino capture and the delayed heating

In the delayed neutrino heating mechanism the energy deposition is through the capture reactions $(N,Z) + \nu \rightarrow (N-1, Z+1) + e^-$ on the nucleus (N,Z) or through a similar reactions for $\bar{\nu}$ with the release of a positron. The captures can be on free protons as well. In a hydrodynamical calculation for a $25M_O$ star (model 25A) due to Wilson [5] the stalled shock first recedes for a while and then beyond about 0.5s due to the heating by $\nu / \bar{\nu}$ it starts moving out leaving a mass of $1.665M_O$ behind. The quasivacuum thus creates is dominated by radiation and holds the matter from falling back. The rate of energy deposition per gram of matter at a radius of R_m in the simple case with only free nucleons present, is given by

$$\dot{E}_+ = \frac{K}{4\pi R_m^2}\left[L_\nu Y_n + L_{\bar{\nu}} Y_p\right] \text{erg g}^{-1}\text{ s}^{-1} \tag{5}$$

where $L_\nu / L_{\bar{\nu}}$ is the $\nu/\bar{\nu}$ luminosity and Y_n and Y_p are the neutron and proton fractions and K is the neutrino absorption coefficient ($K = A\sigma = 9\text{x}10^{-44} <\epsilon_\nu^2> A \text{ cm}^2\text{g}^{-1}$ with A the Avogadro's number and σ the capture cross section). There is , however , loss of energy through the inverse processes (loss rate being $\dot{E}_-$) and assuming the electrons (positrons) form a blackbody gas and using $L_\nu = L_{\bar{\nu}} = 4\pi R_\nu^2 \cdot \frac{1}{4} ac\, T_\nu^4$ (a is the Stefan's constant) one can write for the net heating rate

$$\dot{E} = \dot{E}_+ - \dot{E}_- = K.acT_\nu^4\left[\left(R_\nu/2R_m\right)^2 - \left(T_m/T_\nu\right)^6\right], \tag{6}$$

where T_m is the matter temperature and T_ν and R_ν are the temperature and radius of the neutrinosphere. Eq.(6) shows that for a positive heating the temperature T_m has an upper bound. For the example with $R_\nu = 30$ km, $R_m = 150$ km and $T_\nu = 5$ Mev, $T_m < 2.3$ Mev for $\dot{E}$ to be positive. Bethe and Wilson also considered the corrections to this simple picture by taking into account the energy deposition through the scattering with electrons and loss of energy due to neutrino pair production of all 3 favours. They compared the total internal energy per gram of matter , with GM/R_m assuming the kinetic energy of the matter small. The equation of motion of a mass element is given by

$$\ddot{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} - \frac{GM}{r^2}. \tag{7}$$

Defining $k = -\dfrac{\partial \log p}{\partial \log r}$, one gets

$$r^2\ddot{r} = {}^{krp}\!/_{\rho} - GM. \tag{8}$$

For the simulation of Wilson, $GM \simeq 21 \text{ x } 10^{25}$ cm ^{3}s^{-2}. The product rp/ρ is 2.5-5 x 10^{25}. After the shock recesion, k reaches a high value of 7-9 and this is able to make $\ddot{r}$ positive.

Ray and Kar [20] in a later work considered the heating resulting in the dissociation of the iron-type nuclei simply through $^{56}Fe \rightarrow 13\alpha + 4n$ and $\alpha \rightarrow 2p+2n$. The entropy change due to the heating is evaluated using a TdS equation and the fractions of the four matter species and the temperature are evaluated self consistently by solving the two Saha equations for dissociation alongwith the change in temperature for the entropy change ΔS through the heating. It was found that the success of this scheme depended sensitively on the $\nu/\bar{\nu}$ luminosity.Haxton [21] was the first to point out the importance of neutral current interaction in the inelastic scattering off nuclei. Through this process the μ and the τ neutrinos (having temperatures higher than the electron type neutrino and unaffected by nuclear thresholds) contribute significantly to the heating. A calculation incorporating this heating [22] shows considerable increase in the total internal energy for the $25 M_{\odot}$ resulting in better chances of success for the $25 M_{\odot}$ star but an improvement still unable to achieve success for the $18 M_{\odot}$ star. A recent work by Bruenn and Haxton [23] considered neutrino-nucleus interactions consistently at all stages of the core collapse supernova and made a detailed survey of the contribution from the different heating processes.

6. Some high energy aspects

Alongwith the observation of SN1987A, the neutrino detectors at Kamiokande II (K II) and IMB registered 11 and 8 neutrino events spread over about 12.5 and 5.5 seconds. This is the first time one observed neutrinos simultaneously with a supernova explosion. This induced a lot of activity to find mass limits of electron types neutrinos from the observations. It was based on the assumption that the neutrinos at the source were spread over only a few seconds not 12.5 (as observed for K II) and the delay introduced due to non-zero mass of the neutrinos over the large flight path makes the spread wider at the detector. On the basis of this Kolb *et al* [24] gave a mass limit of about 20 eV whereas Bahcall and Glashow [25] with a somewhat different analysis with only the first 8 events of KII got a mass limit of 11 eV. Spergel and Bahcall [26] carried out a comprehensive statistical treatment of the mass limits based upon an extensive set of Monte Carlo simulations. There were also efforts to probe the effect of neutrino oscillations on this, particularly using the MSW mechanism [27]. This led to zones allowed by SN1987A in the MSW plots of Δm^2 vs $\sin^2 2\theta_\nu / \cos 2\theta_\nu$, like the ones obtained for solar neutrinos. The various other areas where the SN1987A neutrinos gave constraints are neutrino lifetimes, magnetic moments, charges and the masses of the weakly interacting particles proposed in various extension to the standard model.

Recently, Gandhi and Burrows [28] considered the production of sterile (positive helicity) neutrinos via neutral-current neutrino-nucleon scattering in the Weinberg-Salam standard model minimally extended to include Dirac neutrino masses and considered its effect on the energetics of supernova cooling. They found that due to an effect in the cooling rate caused by the sterile neutrinos a value of $m_{\nu_{\mu\tau}} \left(= \sqrt{m_{\nu_\mu}^2 + m_{\nu_\tau}^2} \right)$ larger than about 28 keV is inconsistent with the observation reported in both the detectors . Another calculation by Goswami *et al* [29] considered the effect of massive righthanded Majorana neutrinos on the pre-bounce energy transport and assuming them to be either trapped or free-streaming but not with large enough intensity to prevent the explosion, obtained upper and lower bounds on the right -handed interaction strengths.

Thus, the observations of supernova neutrinos has become a testing ground of various model for neutrino mass generation and their interaction with matter.

Acknowledgments

I thank Dr. Alak Ray for sustained collaboration on the different problems of supernova physics reported here. I also thank Dr. S Sarkar for his contribution in the problem on beta decay rates . I express my gratitude to the Theoretical Physics Seminar Circuit for giving me an opportunity to visit Tata Institute of Fundamental Research, Bombay and Physical Research Laboratory, Ahmedabad to present some of the works reported here.

References

[1] H A Bethe, G E Brown, J Applegate and J M Lattimer, BBAL, 1979 *Nucl. Phys* **A324** 487

[2] G E Brown, H A Bethe and G Baym 1982 *Nucl. Phys.* **A375** 481

[3] J Cooperstein, H A Bethe and G E Brown, 1984 *Nucl. Phys* **A429** 527

[4] E A Baron, H A Bethe, G E Brown, J Cooperstein and S Kahana 1987 *Phys. Rev. Lett.* **59** 736 ; E A Baron, J Cooperstein and S Kahana 1985 *Nucl. Phys.* **A440** 744

[5] R Wilson 1985 *Numerical Astrophysics* eds J Centrella, J Le Blane and R Bowers (Boston: Jones and Bartlett) p422.

[6] H A Bethe and J R Wilson 1985 *Astrophys.J* **295** 14

[7] K Nomoto,T Shigeyama, S Kumagai and H Yamaoka 1991 *Supernovae and Stellar Evolution* eds A Ray and T Velusamy (Singapore : World Scientific) p116

[8] B A Brown and B H Wildenthal 1985 *At Data and Nucl. Data Tables* **33** 347

[9] G M Fuller, W A Fowler and M J Newman [FFN] 1980, *Astrophys. J* **42** 447, 1982a *Astrophys. J (Suppl.)* **48** 279, 1982b *Astrophys.J.* **252** 719

[10] H A Bethe 1990 *Rev Mod. Phys.* **62** 801

[11] M B Aufderheide, G E Brown, T T S Kuo, D B Stout and P Vogel 1990 *Astrophys. J.* **362** 241

[12] K Kar, S Sarkar and A Ray 1991 *Phys Lett.* **B261** 217, Errata 1992 *Phys Lett.* **B277** 528

[13] J B French and V K B Kota 1982 *Ann. Rev Nucl Part. Sci* **32** 35

[14] V K B Kota and K Kar 1989 *Pramana* **32** 674

[15] G M Fuller 1982 *Astrophys J* 252

[16] A Ray, S M Chitre and K Kar 1984 *Astrophys J* **285** 766

[17] K Kar and A Ray 1983 *Phys. Lett.* **B96** 322

[18] J Cooperstein and J Wambach 1984 *Nucl Phys.* **A420** 591

[19] S W Bruenn 1985 *Astrophys.J.* **58** 143

[20] A Ray and K Kar 1987 *Astrophys. J.* **319** 143

[21] W C Haxton 1988 *Phys. Rev. Lett.* **60** 1994

[22] A Ray and K Kar 1992 *Astrophys. Lett. Comm.* **28** 271

[23] S W Bruenn and W C Haxton 1992 *Astrophys. J.* **376** 678

[24] E W Kolb, A J Stebbins and M S Turner 1987 *Phys. Rev.* **D35** 3598

[25] J N Bahcall and S L Glashow 1987 *Nature* **326** 1476

[26] D N Spergel and J N Bahcall 1987 *Phys. Lett.* **B200** 366

[27] D Nötzold 1987 *Phys. Lett.* **B196** 315

[28] J Gandhi and A Burrows 1990 *Phys. Lett.* **B246** 149

[29] S Goswami, K Kar and A Raychaudhuri 1992 *Phys. Rev.* **D46** 3278

[30] K Takahashi, J M Yamada and J T Kondoh 1973 *At. Data. Nucl. Data Tables* **12** 101

[31] H V Klapdor, J Metzynger and T Oda 1990 *At. Data. Nucl. Data Tables* **44** 73

[32] K Kar, S Sarkar and A Ray 1993(unpublished)